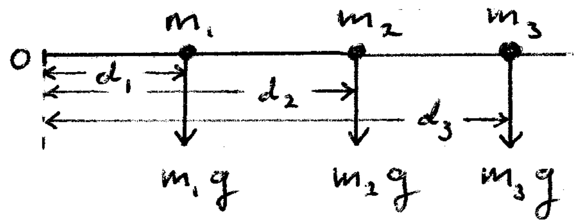
Mechanics Examples Sheet 3 – Solutions

1. To find the c.o.m. take moments at O. This is equivalent to using the formula

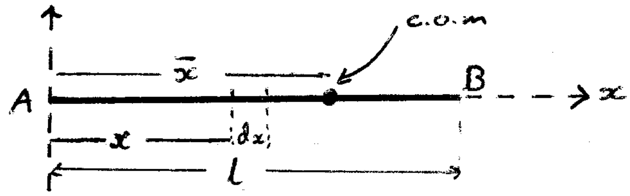


$$dM = \sum_i d_i m_i ,$$

where d is the distance between the origin, O, and the c.o.m. Thence,

$$d = \frac{d_1 m_1 + d_2 m_2 + d_3 m_3}{m_1 + m_2 + m_3}$$

2.



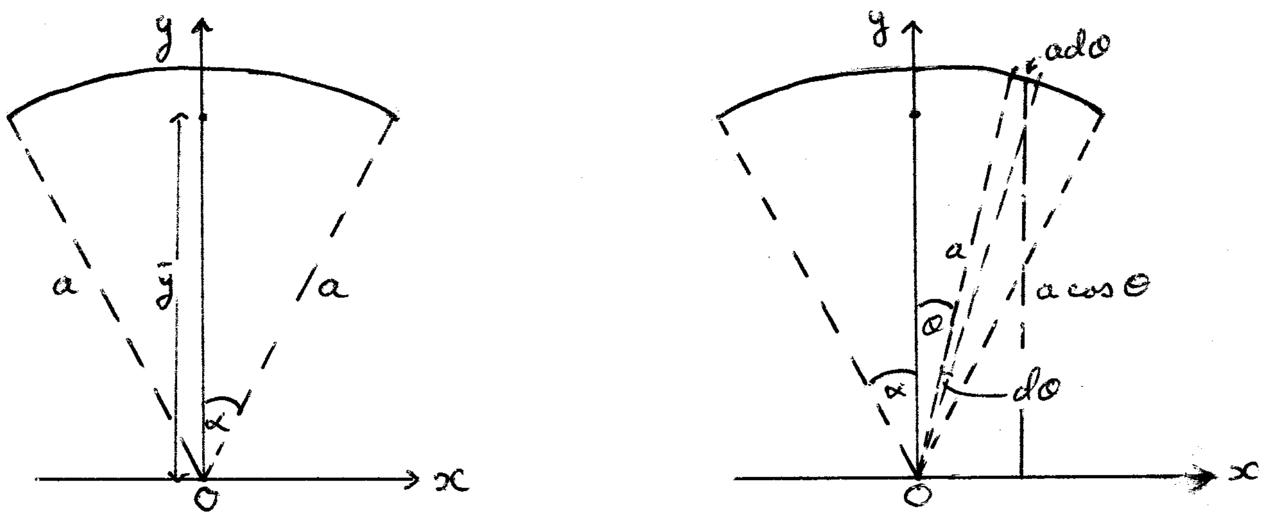
$$\text{Mass} = \int_0^l \frac{\mu x}{l}\, dx$$

$$= \frac{1}{2}\mu l$$

Let the position of the c.o.m. be $\bar{x}$. Then

$$\bar{x} = \frac{\int_0^l \frac{\mu x}{l} . x\, dx}{\frac{1}{2}\mu l} = \frac{2}{3} l$$

3.



Devide the arc into smaller arcs subtending an angle $d\theta$ at its centre O so that each smaller arc of length $a\,d\theta$ can be thought of as the apparent 1-D volume of a particle. If ρ is the density per unit length then each particle has mass $\rho a\,d\theta$. On taking moments about the x-axis:

$$\bar{y} M = \sum_i m_i y_i$$

$$\Rightarrow \bar{y} \int_{-\alpha}^{\alpha} \rho a \, d\theta = \int_{-\alpha}^{\alpha} (\rho a \, d\theta \cdot a \cos\theta)$$

or

$$\bar{y} = \frac{\int_0^{\alpha} \rho a^2 \cos\theta \, d\theta}{\int_0^{\alpha} \rho a \, d\theta} = \frac{a \sin\alpha}{\alpha}$$